LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2014**

ST 2814/2811 - ESTIMATION THEORY

Date : 28/03/2014 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

Part – A

Answer ALL the following: (10 X 2 =20)

1) Define UMVUE for estimating a parameter θ.
2) Suggest an unbiased estimator of θ, when a random sample $X_1, X_2, \ldots, X_n$ is drawn from U(0, θ).
3) Obtain the sufficient statistic when a random sample $X_1, X_2, \ldots, X_n$ is drawn from
   $P(x,\theta) = \theta x^{\theta-1}$, 0 < X < 1, zero elsewhere.
4) Find which one of the following is ancillary when a random sample X_1, X_2 is drawn from N(μ,1)
   (a) X_1/X_2 (b) X_1+X_2 (c) X_1 - X2 (d) $2X_1-X_2$
5) Define bounded completeness.
6) Define a minimal sufficient statistic.
7) State the sufficient condition for an estimator to be consistent.
8) Define mean square error. What is the mean square error of $\bar{X}$ when the random sample is drawn from $N(\mu,\sigma^2)$?
9) State any two Rao – Cramer regularity conditions.
10) Suggest an MLE for P[X=0] in the case of Poisson (θ).

PART – B

Answer any FIVE questions: (5x8 =40)

11) Let δ_0 be a fixed member of U_g. prove that $U_g = \{ \delta_0 + u \mid u \in U_0 \}$.
12) Let $X_1, X_2, \ldots, X_n$ be a random sample from E(μ,σ). Obtain the MLE of μ and σ.
13) Let X ~ N(θ,1). Obtain the Cramer-Rao lower bound for estimating θ^2. Compare the variance of the UMVUE with the Cramer-Rao lower bound.
14) State and prove the invariance property of the CAN estimator.
15) Let $X_1, X_2, \ldots, X_n$ be iid Poisson (λ) where λ ~ E(0,1). Find the Baye's estimator of λ.
16) Obtain the minimal sufficient statistic in the case of b(1,θ) based on a random sample.
17) Let X be a discrete r.v with $p(x,\theta) = \begin{cases} \theta & x = -1 \\ (1-\theta)^2\theta^x & x = 0, 1, 2, \ldots \end{cases}$

    Find all the unbiased estimators of 0.
18) Let X_1, X_2 be a random sample from E(0,σ). Show that $(X_1 + X_2)$ and $X_1 \mid (X_1 + X_2)$ are independent using Basu's theorem.

PART – C

Answer any TWO of the following: (2 X 20 = 40)

19) (a) State and prove Rao-Blackwell theorem. Hence obtain Lehman-Scheffe theorem.
(b) Show that the UMVUE is unique.
(c) Let $X_1, X_2, \ldots, X_n$ be a random sample from Poisson (θ). Obtain the a UMVUE of $e^{-\theta}$.

20) (a) State and prove Cramer-Rao inequality for multiparameter case.
(b) Obtain the Cramer-Rao lower bound for i) μ and ii) σ^2 when the random sample is from $N(\mu,\sigma^2)$.

21) (a) State and prove the small sample properties of the MLE.
(b) Let $X_1, X_2, \ldots, X_n$ be iid $E(\theta,1)$. Show that the MLE of θ is not CAN but consistent. Suggest a CAN estimator for θ.

22) (a) Explain Jacknife estimator with an example.
(b) Explain EM algorithm in detail.

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